LAU
School of Arts and Sciences
Department of Computer Science & Mathematics

MTH207 – Discrete Mathematics
Spring 2013-2014
Exam 2
(April 25, 2014)

Name: ID:

Duration: 60 minutes

Instructor: Silvana Nahlus

- Answer the questions in the space provided for each problem.
- If more space or scratch is needed, you may use the back pages.
- Only scientific calculators are permissible.
- The exam has **5** pages consisting of **7** exercises.

Grades:

1.a 8%	
1.b 8%	
2. 10%	
3.a 10%	
3.b 6%	
3.c 10%	

4. 12%	
5. 12%	
6. 12%	
7. 12%	
Total 100%	

1. a) Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

   b) Show that $A - (B \cup C) = (A - B) \cap (A - C)$.

2. Given that the sets $A_1,\ A_2,\ A_3, \dots \dots$ are all countable, show that $\bigcup_{i=1}^{\infty} A_i$ is also countable.

3. a)Give as good a big-O estimate as possible for the function $f(x) = (\log n! + \sqrt{n})(n^2 + 2)$

b)Define the statement $f(x, y)$ is $\Theta(g(x, y))$, that is define $f(x, y)$ is a big theta of $g(x, y)$.

c)Let k be a positive integer. Show that $1^k + 2^k + \ldots\ldots + n^k$ is $O(n^{k+1})$

4. If a and r are real numbers and $r \neq 0$, then **prove**

$$\sum_{j=0}^{n} \boldsymbol{a\, r^j} = \begin{cases} \dfrac{\boldsymbol{ar^{n+1} - a}}{\boldsymbol{r-1}} & \boldsymbol{if\ r \neq 1} \\ \boldsymbol{(n+1)a} & \boldsymbol{if\ r = 1} \end{cases}$$

5. Find the Boolean product of A and B^T, where $A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ and

$B = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$

6. Let $f: Z \times Z \to Z \times Z$ and $g: Z \times Z \to Z \times Z$ be defined by $f(m, n) = (m + 2, n - 3)$ and $g(m, n) = (m - 2, n + 3)$. Show that $f \circ g$ is one-to-one.

7. Find a formula for $\sum_{k=0}^{m^2-1} \lfloor \sqrt{k} \rfloor$, where m is a positive integer.